[P17] [Every pure bipartite entangled state is nonlocal] (4)

The aim of this exercise is to prove that any bipartite and pure entangled state is also nonlocal. That is, any state $|\Psi\rangle_{AB} \neq |\Psi\rangle_A \otimes |\Psi\rangle_B$ violates a Bell inequality. We will restrict our attention to the case of qubits, that is, the single systems are described by 2-dimensional Hilbert spaces. To prove that we will use the Horodecki's criterion stating that for a two-qubit state ρ_{AB} the maximum value of the CHSH inequality is given by

$$CHSH = 2\mathcal{M}_{\mathrm{CHSH}}(\rho_{AB}) = 2\sqrt{t_{11}^2 + t_{22}^2}, \tag{1}$$

being t_{11}^2 and t_{22}^2 the two largest eigenvalues of $\mathcal{T}^\dagger \mathcal{T}$, with $\mathcal{T}_{i,j} = \mathrm{tr}\left[(\sigma_\mathrm{i} \otimes \sigma_\mathrm{j})\, \rho_\mathrm{AB}\right]$, where σ_i refers to the Pauli matrices with $1 \leq i \leq 3$.

(1) Prove that local unitaries $U_A \otimes U_B$ cannot create entanglement, that is, they map separable states to separable states.

(2) For pure states the Peres criterion discussed in class is a necessary and sufficient condition for the existence of entanglement. Using that, show that any two-qubit state of the form $|\Psi\rangle_{AB} \neq |\Psi\rangle_A \otimes |\Psi\rangle_B$ is entangled

(3) Using the Horodecki's criterion, prove that any pure two-qubit state is nonlocal.

Hint: For (2) and (3), use the Schmidt decomposition and the result in (1) to simplify the form of the general states $|\Psi\rangle_{AB} \neq |\Psi\rangle_A \otimes |\Psi\rangle_B$.

[P18] [Not every entangled state violates the CHSH inequality] (4)

Consider the so called Werner state given by $\rho_W(v) = v|\Phi^-\rangle\langle\Phi^-| + (1-v)\mathbb{I}/4$, with $|\Phi^-\rangle = (1/\sqrt{2})\left(|0\rangle \otimes |0\rangle - |1\rangle \otimes |1\rangle\right)$.

(1) Using that the Peres criterion is a necessary and sufficient condition for two-qubit states, compute the values of v for which the Werner state is entangled.

(2) Use the Horodecki criterion to compute the values of v for which the Werner state violates the CHSH inequality.