[P2] In the lecture, we have sketched a way of casting the statement "a specific Turing machine T will halt eventually when acting on input x" as a statement about natural numbers. Here, we will have a closer look at this trick.

The Turing machine T is defined, as usual, by the data $\langle Q, \Sigma, \delta, q_0, F\rangle$. We assume that the elements of the tape alphabet Σ have been numbered in some way from 0 to $|\Sigma| - 1$ and that the same is true of the elements of Q.

As defined in the lecture, the configuration of the Turing machine at a specific point of time will be encoded by a bit string y. The first $\lceil \log_2 |Q| \rceil$ bits of y are used to store the binary representation of the number of the internal state $q \in Q$. The following $\lceil \log_2(|\Sigma| + 1)\rceil$ bits contain the (binary representation of) the number of the symbol $\sigma_1 \in \Sigma$ directly underneath the head of the TM. This is followed by the number of the symbol σ_2 directly to the right of the head, and so on, based on the scheme used in the lecture. Finally, we store the number $|\Sigma| + 1$ to indicate that all further tape cells are blank. Let

$$n(y) = \sum_{i=0} y_i 2^i$$

be the natural number defined by the bit string $y = \langle y_0, y_1, \ldots \rangle$.

(1) In this first problem, we use the specific Turing machine of the first lecture. Its states are $Q = \{\mathrm{SR}, \mathrm{SL}, \mathrm{C}, ☺, ☹\}$ assumed to be number in this order from 0 to 4. The alphabet is $\Sigma = \{(,), \mathrm{E}, \mathrm{X}, \text{\_}\}$ (where the finally symbol '␣' indicates a blank cell). Again, these are labeled from 0 to 4. In the lecture, we encountered the following configuration:

$$\begin{array}{cccccccccccccc} & & & & & \nabla & & & & & & & & \\ \ldots & \text{\_} & \text{\_} & \mathrm{E} & (& \mathrm{X} & \mathrm{X} & (&) &) & \mathrm{E} & \text{\_} & \text{\_} & \ldots \end{array}$$

with internal state $q = \mathrm{SR}$. Convert this into a bit string y (please explain what you're doing). Give the decimal representation of $n(y)$ (please use a computer). (4 P.)

(2) Let s_1 be a number between 1 and $|Q|$. In the sketched definition of the formula $\textsc{Valid}(n_1, n_2)$, we made use of a formula $\textsc{State}(n_1, s_1)$ defined to be true if and only if the number of the internal state q of the configuration represented by the number n_1 is equal to s_1. Write out the definition of $\textsc{State}(n_1, s_1)$ using only the symbols introduced in the lecture, and maybe the formulas $\textsc{Bit}(n, i)$ and $\textsc{Compare}(n, m, i, j)$. (2 P.)